UNIVERSITÀ DEGLI STUDI DI MILANO
DIPARTIMENTO DI FISICA

Levitation, oscillation and wave propagation in a stratified fluid

Marina Carpineti
Physics Department seminar – November 12th 2021

Alberto Vailati and Fabrizio Croccolo

https://www.esa.int/Science_Exploration/Human_and_Robotic_Exploration/In_search_of_stable_liquids https://technes.fisica.unimi.it

TechNES Space project: an international research team coordinated by UNIMI and funded by ESA with an industrial partner is working to investigate the behavior of complex fluids in space, with a focus on developing new technologies for pharmaceutical and biomedical fields, based on non-equilibrium fluctuations. In particolar, we will use non-equilibrium fluctuations as a tool to measure relevant thermo-physical properties of complex fluids.

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universität wien

NanoTemper

Overcome gravity

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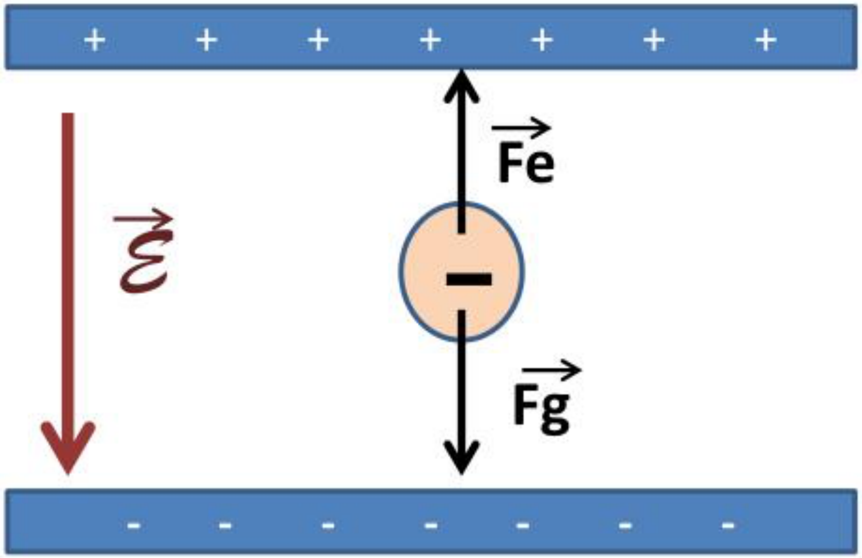
Overcome gravity



Millikan experiment

On the elementary electrical charge and the Avogadro constant
R.A. Millikan Phys Rev **2** 109 (1913)

Nobel Prize 1923

Levitation

Many interesting physical phenomena give rise to true levitation.

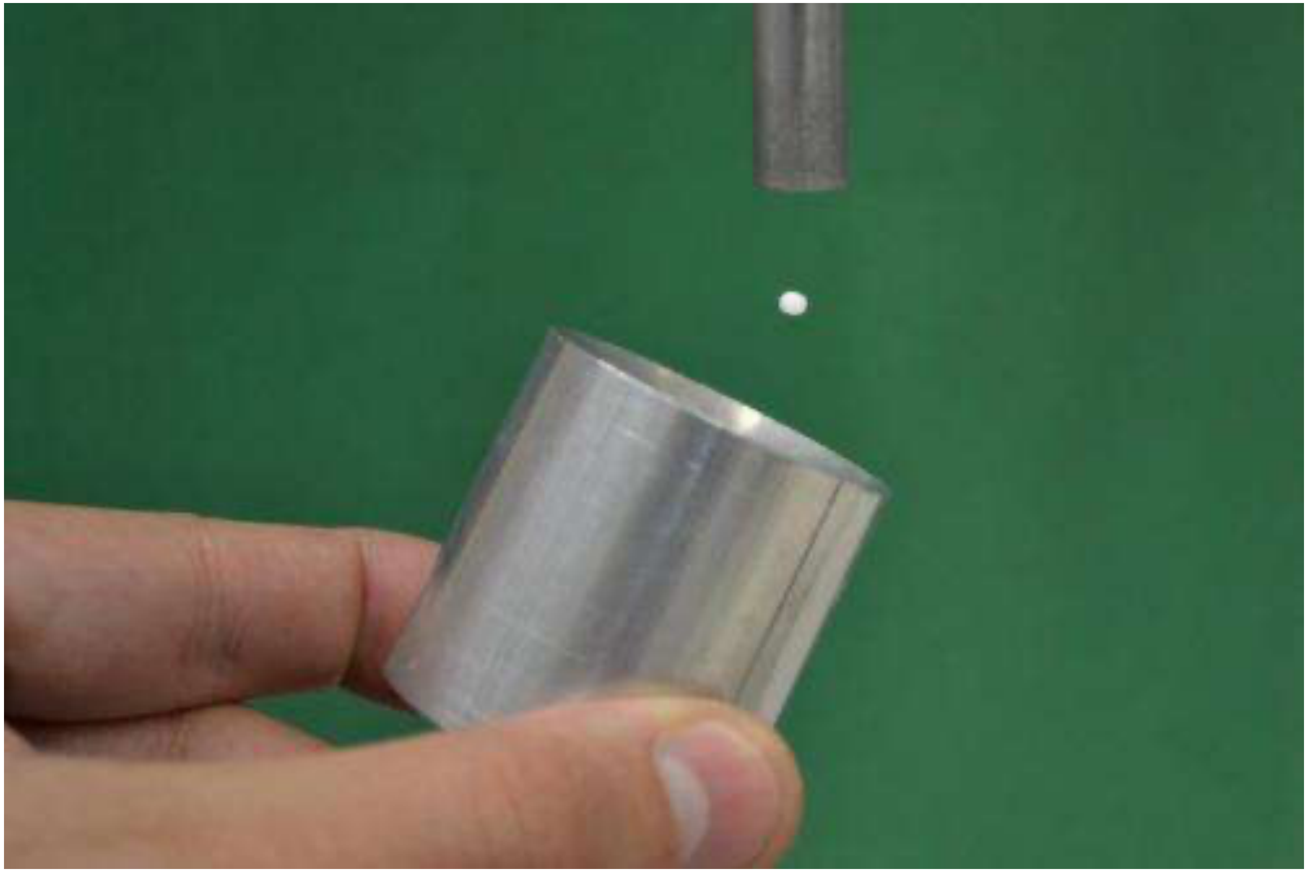
Not only forces with action at a distance, but stable equilibrium.

A restoring force brings the body back to its position when it is slightly moved.

« Levitation in Physics» Brandt, E H
Science; Jan 20, 1989; 243, 4889

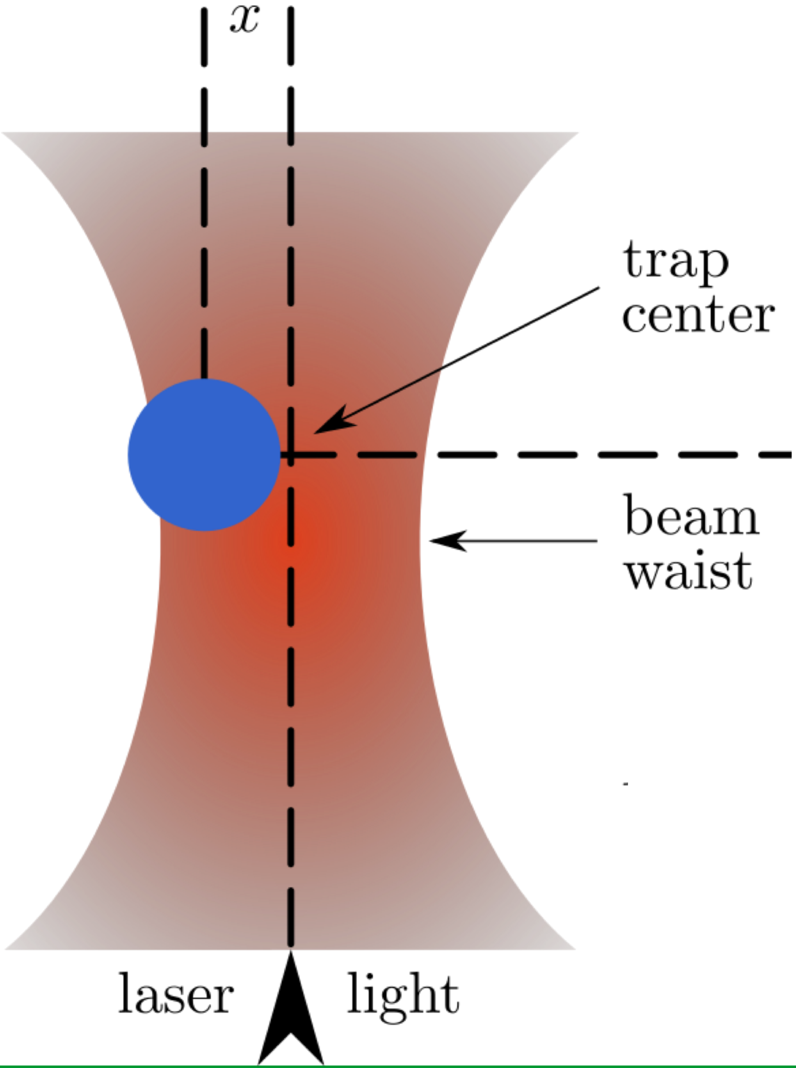
Acoustic levitation

Acoustic levitation



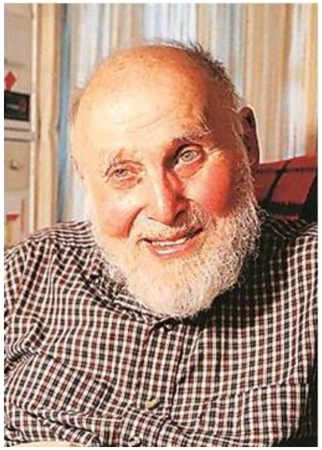
Tiny Lev: A multi-emitter single-axis acoustic levitator;
Marzo et al. Rev. Sci. Instr. **88**, 085105 (2017)

Optical levitation



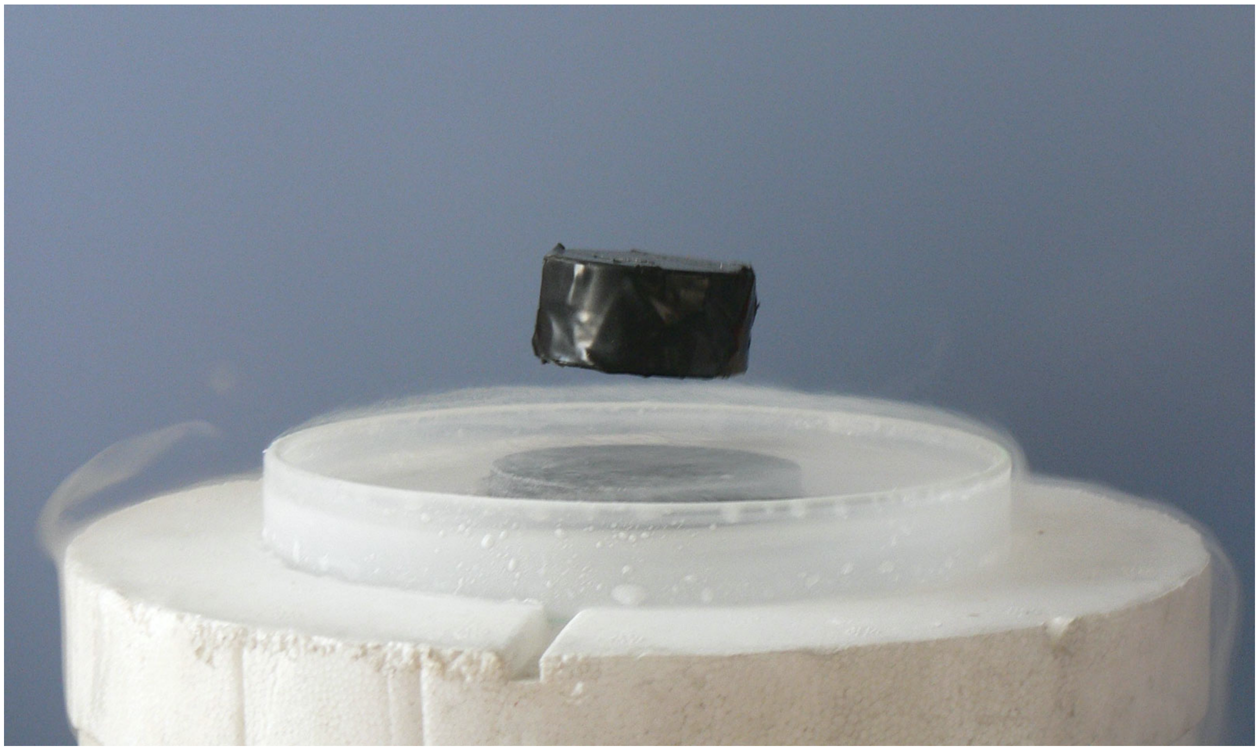
Optical tweezer

Acceleration and trapping of particles by radiation pressure
A. Ashkin ; Phys. Rev. Lett. **24,** 156 (1970)



Nobel Prize 2018
for the optical tweezers and their application to biological systems

Magnetic levitation



Superconductivity

Nobel Prize 1913
(Kamerlingh- Onnes)

Nobel Prize 1972
(Bardeen,Cooper and Schriefer)

Nobel Prize 1987
(Bednorz and Müller)

for their important break-through in the discovery of superconductivity in ceramic materials.

Magnetic levitation

On flying frogs and levitrons

M.V. Berry and A.K. Geim
Eur. J. Phys., 18 (1997) 307-313

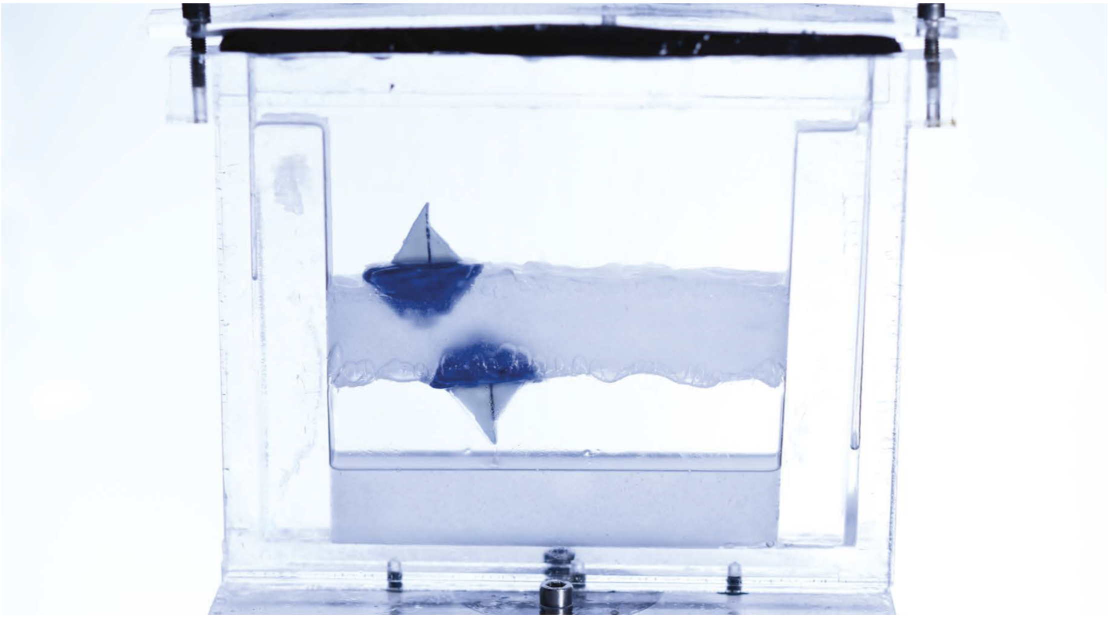
IgNobel Prize 2000

Inquiry based learning

Levitation is a source of inspiration for scientists for the challenging experiments that involve it, and for its technological application, but in particular for the surprise it creates in a world dominated by gravity.

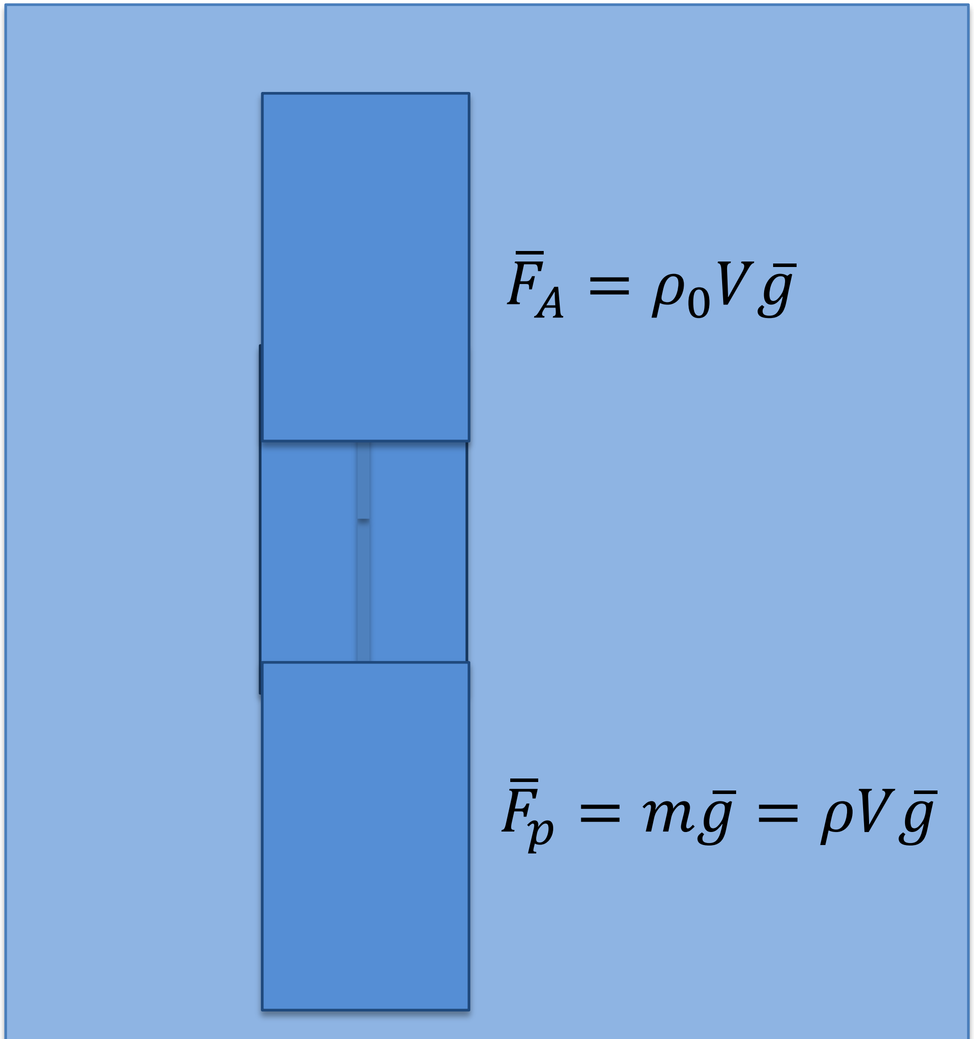
Good candidate for an inquiry based learning

Engagement and surprise stimulate
- curiosity
- questions
- search of answers
- scientific method



B. Appfel et al Nature | Vol 585 | 3 September 2020

Buoyancy



$$F_T = (\rho - \rho_0)Vg$$

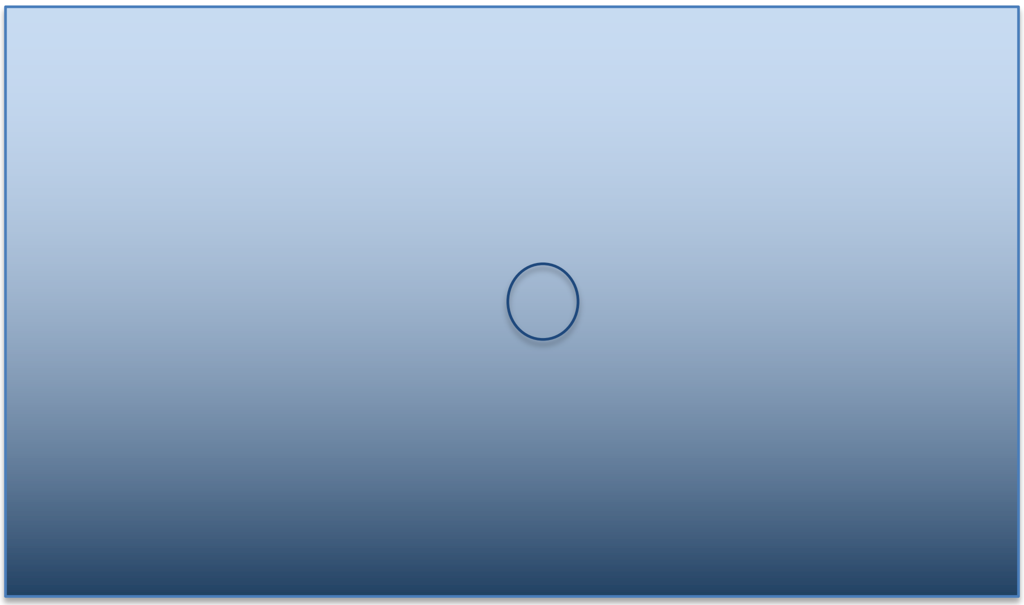
$F_T = 0$ when $\rho = \rho_0$
= neutral buoyancy

Neutral buoyancy

Neutral buoyancy

Nautilus

Equilibrium in a stratified fluid



A stable equilibrium condition may be achieved in a stratified compressible fluid as a gas stratified by gravity.

Gravitationally stable density stratification in a liquid under non equilibrium condition

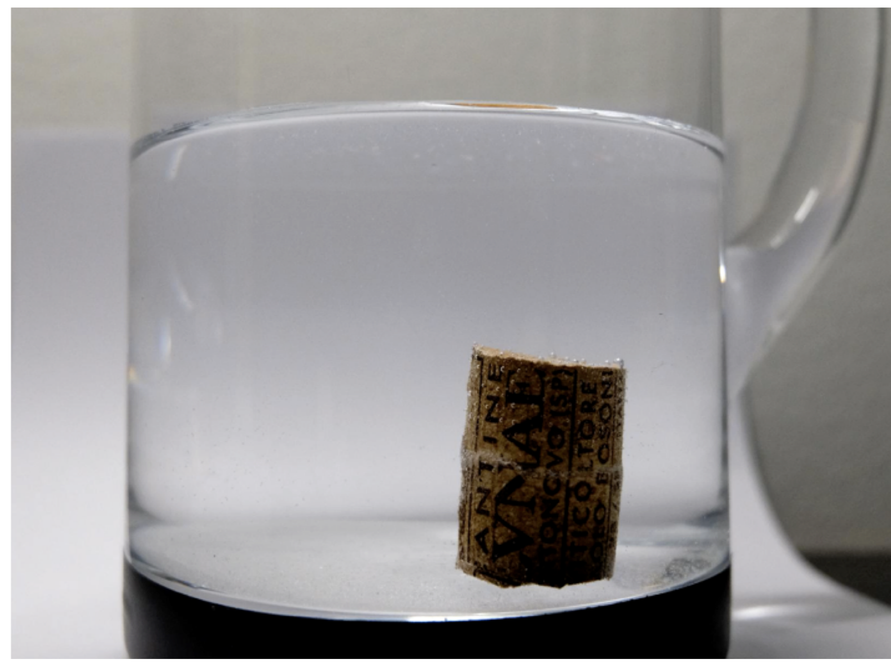
$$\nabla\rho = \frac{\partial\rho}{\partial T}\,\nabla T$$

$$\nabla\rho = \frac{\partial\rho}{\partial c}\,\nabla c$$

$$\nabla\rho = \rho(\alpha\nabla T + \beta\nabla c)$$

Magic cork

We weighted down a cork of a wine bottle using a steel marble



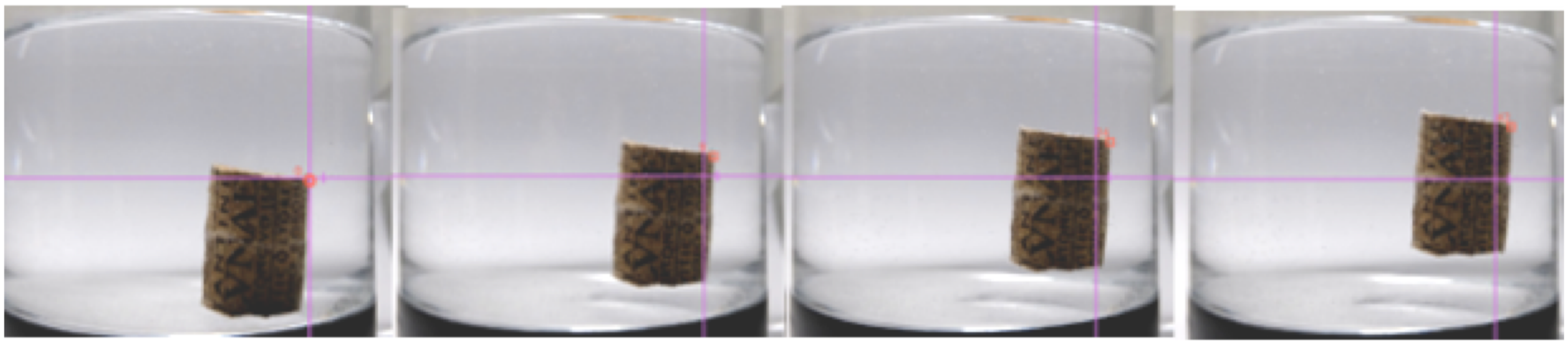
Carpineti, Croccolo and Vailati 2021 *Eur. J. Phys.* **42** 055011

Sinking cork

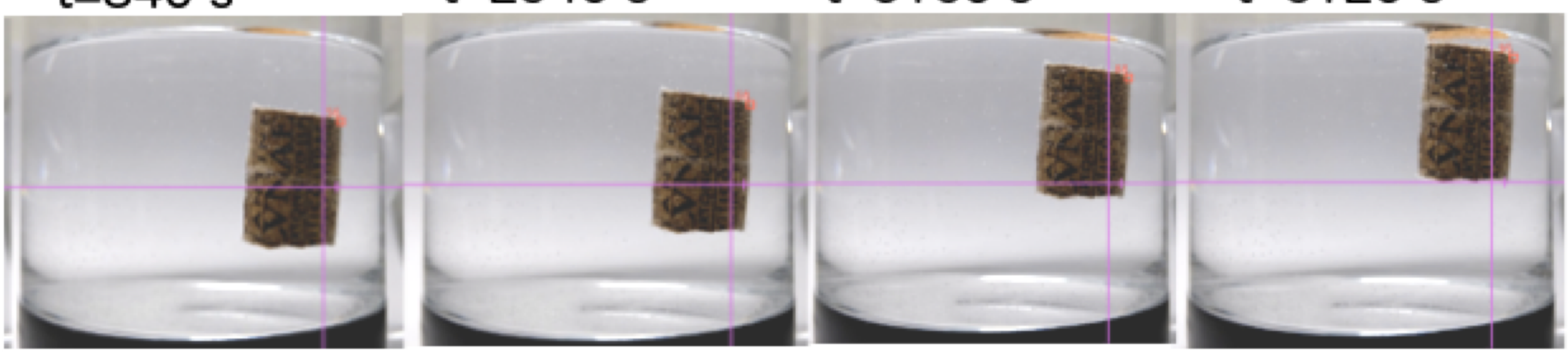
Conc. NaCl
from 10 to 46 g/l

$\rho > \rho_0$

Tracking the cork



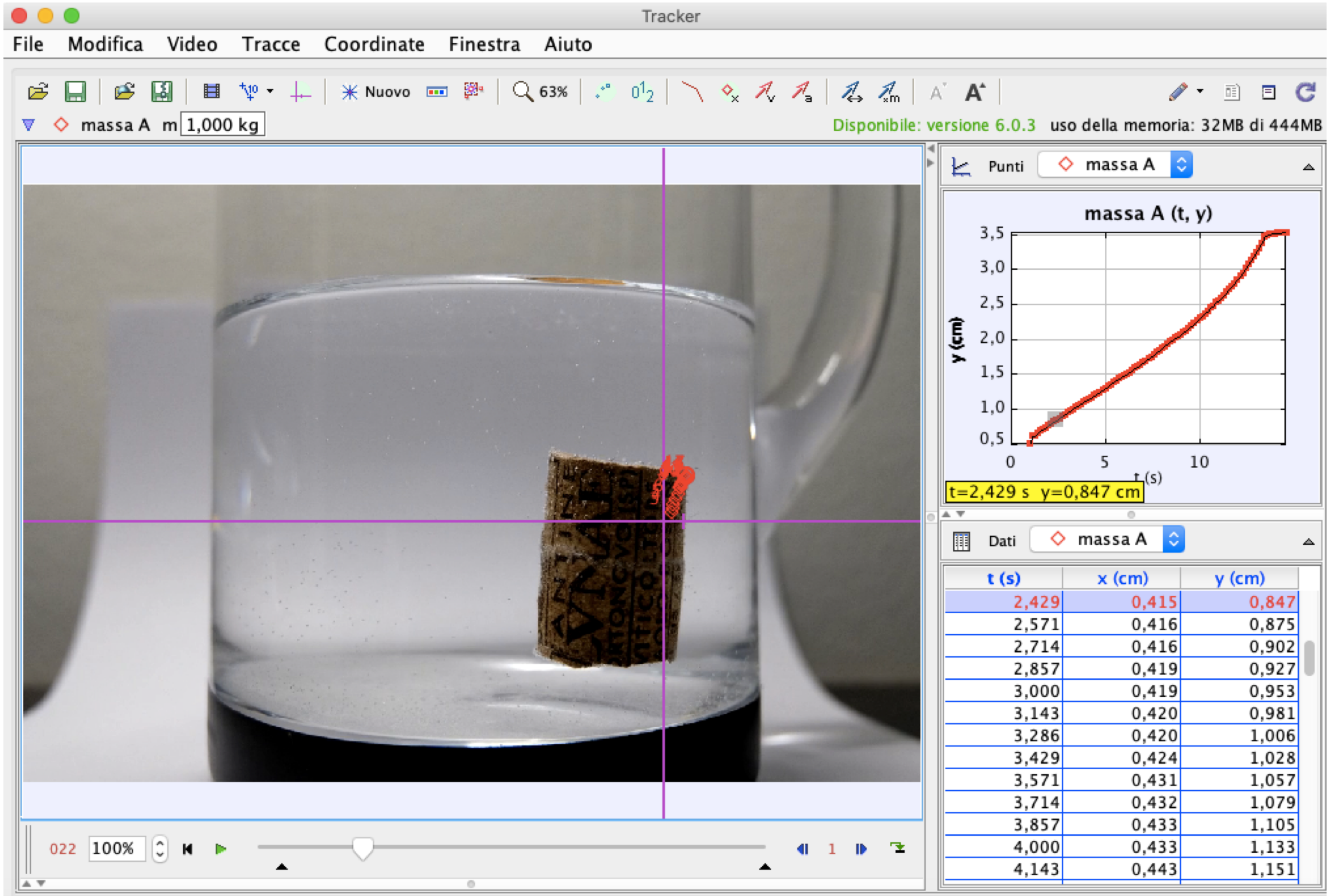
t=840 s t=2940 s t=5160 s t=6120 s



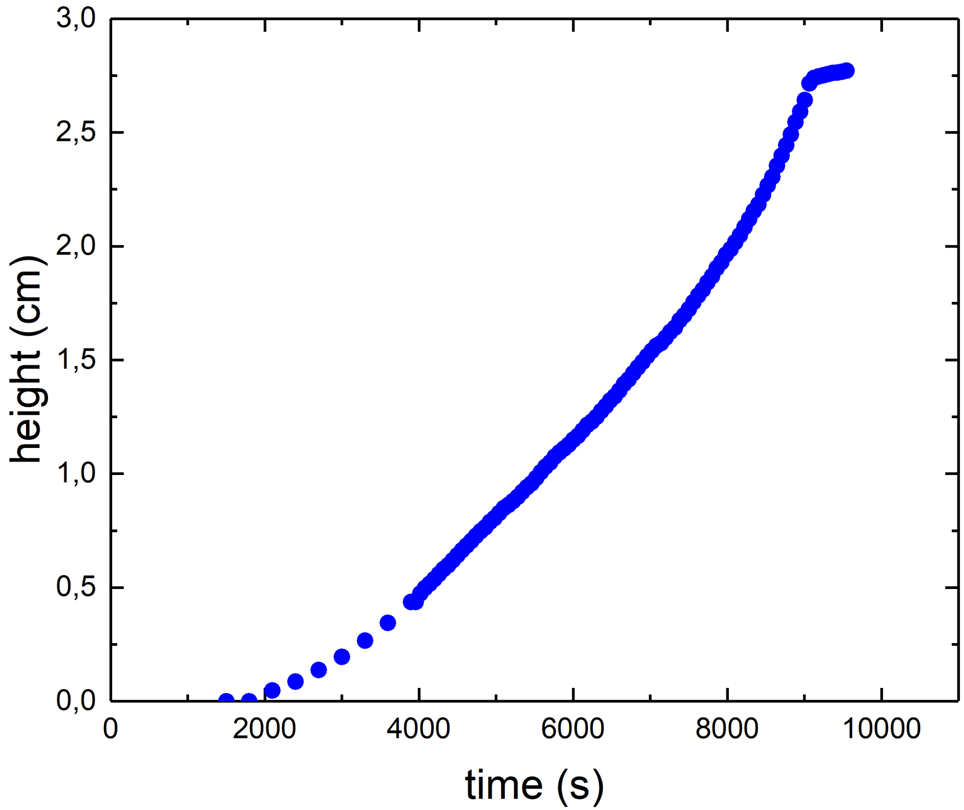
t=7140 s t=7800 s t=9000 s t=9420 s

Tracker

Video Analysis and Modeling Tool



Cork's rise



In principle the cork rises following the evolution of the concenration profile that can be obtained fro the diffusing equations with the proper boundary conditions

$$\frac{\partial c}{\partial t} = D\nabla^2 c$$

Initial phases

Oscillations

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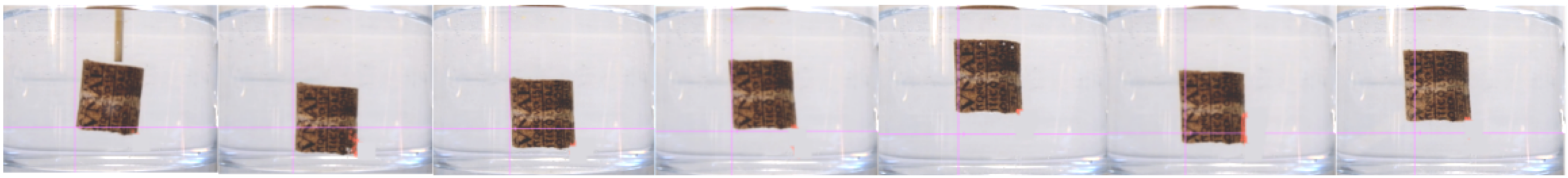
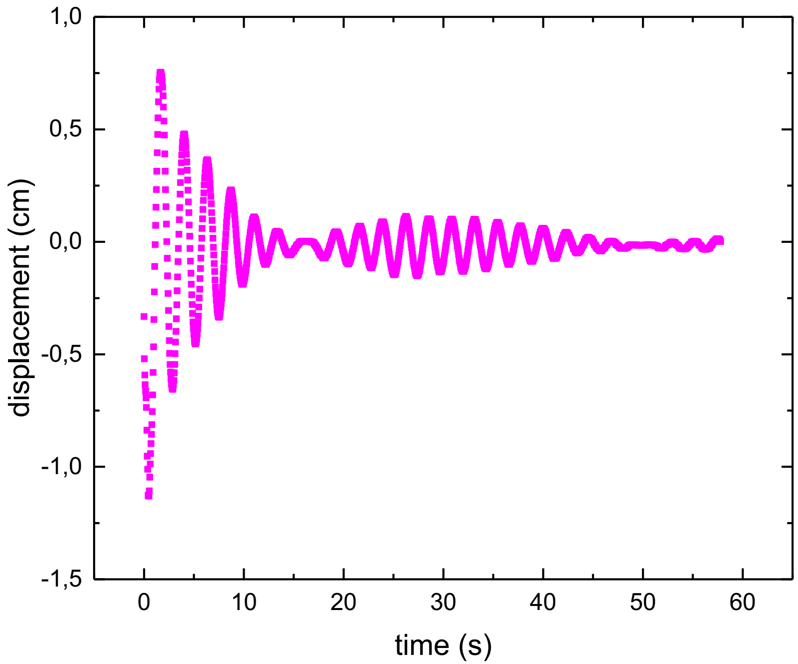
Tracking of the oscillations





Oscillations in a stratified fluid

$$\nabla p = \rho g \qquad \rho(z) = \rho_0 + \frac{\partial \rho}{\partial z} z$$

Buoyancy force acting on a volume V_0:

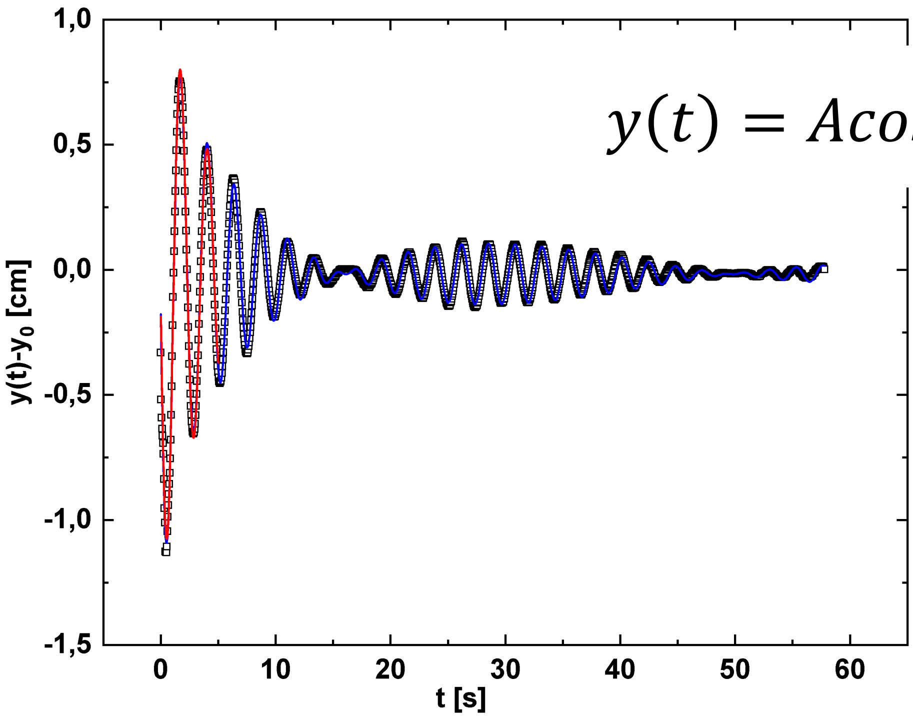
$$F_B = -\left| \oint_S p(z) d\boldsymbol{S} \right| = \int_{V_0} \frac{dp}{dz} dV = \rho_0 g V_0 + \frac{\partial \rho}{\partial z} g V_0 z$$

$$F_T = \frac{\partial \rho}{\partial z} g V_0 z \qquad \ddot{z} = \frac{g}{\rho_0} \frac{\partial \rho}{\partial z} z$$

Brunt-Väisälä (buoyancy) frequency → $N = \sqrt{-\frac{g}{\rho_0}\frac{\partial \rho}{\partial z}}$

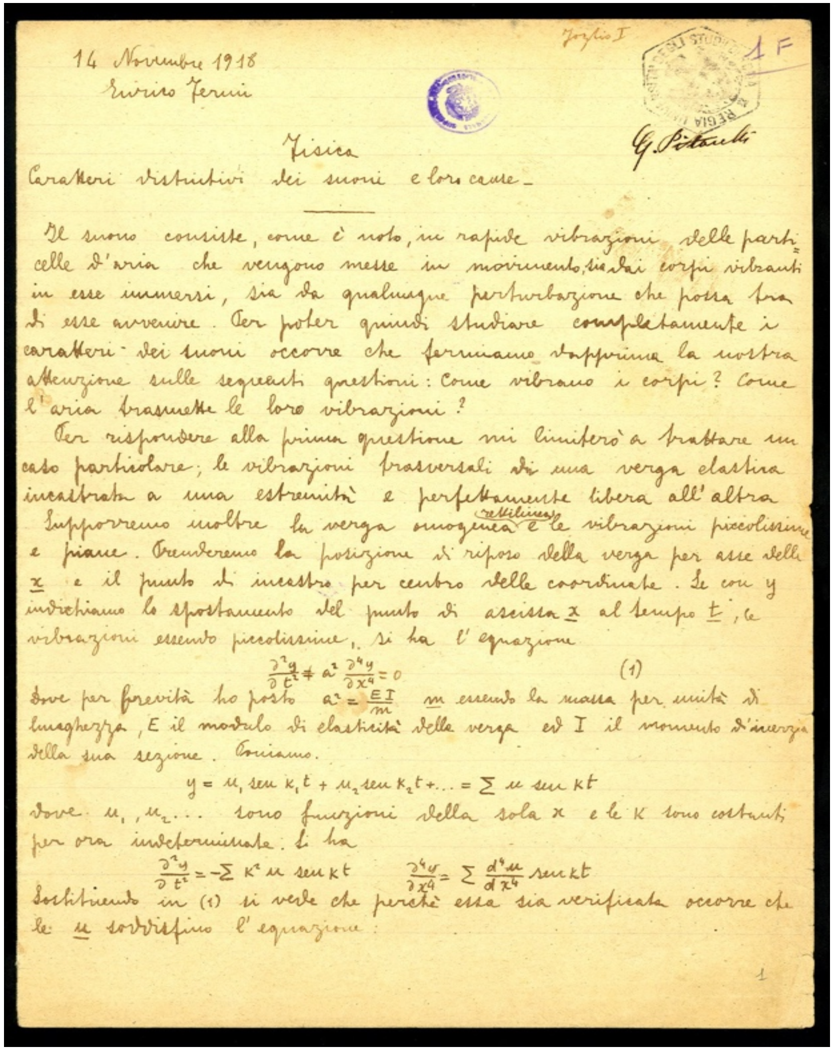
Fit of the oscillations

We can fit the first oscillations with the formula for the damped harmonic oscillations:



$$y(t) = A cos(\omega t) \exp\left(-\frac{t}{\tau}\right) + y_0$$

Origin of the beatings



Foglio I

1 F

14 Novembre 1918
Enrico Fermi

G. Pitanti

Fisica

Caratteri distintivi dei suoni e loro cause –

Il suono consiste, come è noto, in rapide vibrazioni delle particelle d'aria che vengono messe in movimento, sia dai corpi vibranti in esse immersi, sia da qualunque perturbazione che possa tra di esse avvenire. Per poter quindi studiare completamente i caratteri dei suoni occorre che fermiamo dapprima la nostra attenzione sulle seguenti questioni: Come vibrano i corpi? Come l'aria trasmette le loro vibrazioni?

Per rispondere alla prima questione mi limiterò a trattare un caso particolare; le vibrazioni trasversali di una verga elastica incastrata a una estremità e perfettamente libera all'altra. Supporremo inoltre la verga omogenea rettilinea e le vibrazioni piccolissime e piane. Prenderemo la posizione di riposo della verga per asse delle x e il punto di incastro per centro delle coordinate. Se con y indichiamo lo spostamento del punto di ascissa x al tempo t, le vibrazioni essendo piccolissime, si ha l'equazione

$$\frac{\partial^2 y}{\partial t^2} + a^2 \frac{\partial^4 y}{\partial x^4} = 0 \qquad (1)$$

dove per brevità ho posto $a^2 = \frac{EI}{m}$, m essendo la massa per unità di lunghezza, E il modulo di elasticità della verga ed I il momento d'inerzia della sua sezione. Poniamo.

$$y = u_1 \operatorname{sen} k_1 t + u_2 \operatorname{sen} k_2 t + \ldots = \sum u \operatorname{sen} kt$$

dove $u_1, u_2 \ldots$ sono funzioni della sola x e le k sono costanti per ora indeterminate. Si ha

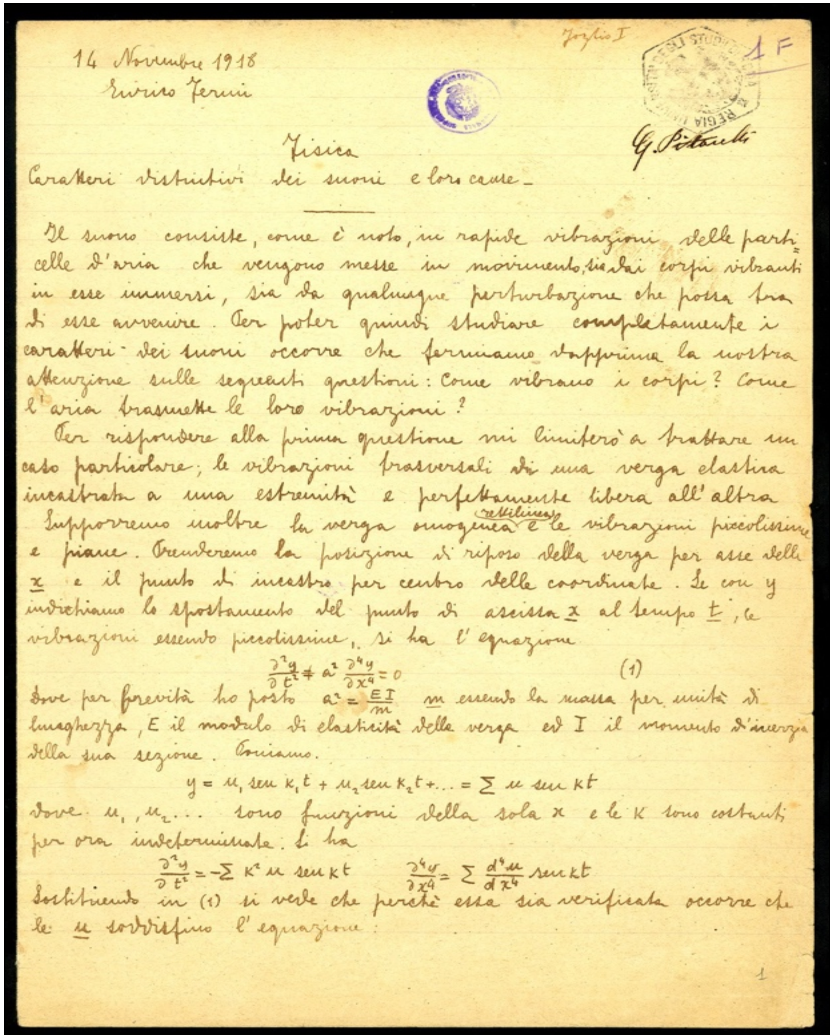
$$\frac{\partial^2 y}{\partial t^2} = -\sum k^2 u \operatorname{sen} kt \qquad \frac{\partial^4 y}{\partial x^4} = \sum \frac{d^4 u}{dx^4} \operatorname{sen} kt$$

Sostituendo in (1) si vede che perchè essa sia verificata occorre che le u soddisfino l'equazione:

1

14 November 1918
Enrico Fermi (17 years old) writes an essay for the admission to Pisa Normale University:
"Distinctive characteristics of sounds and their causes"
("Caratteri distintivi dei suoni e loro cause")

Origin of the beatings



Foglio I

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Enrico Fermi

G. Pittarelli

Fisica

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1

How do bodies vibrate?
How does the surrounding medium* transfer their vibrations?

*air in case of Fermi example

Hydrodynamic equations

In an incompressible inviscid fluid with density stratification the Eulerian hydrodynamic equations are:

$$\nabla \cdot \bar{u} = 0$$ ← continuity equation

$$\rho' \frac{\partial \bar{u}}{\partial t} + \rho' \bar{u} \cdot \nabla \bar{u} = -\nabla p' + \rho' \bar{g}$$ ← second law of dynamics

$$\frac{\partial \rho'}{\partial t} + \bar{u} \cdot \nabla \rho' = 0$$ ← non compressible fluid

$$p' = p + \delta p$$

$$\rho' = \rho + \delta \rho$$

Wave propagation

In the hypothesys of small perturbations $\left|\frac{\delta p}{p}\right| \ll 1$ $\left|\frac{\delta \rho}{\rho}\right| \ll 1$

the equations may be linearized in **u**, δp and $\delta \rho$ and the results are:

- ✓ the velocity is perpendicular to the wave vector (transverse wave)
- ✓ the pressure doesn’t propagate (no emission of sound)
- ✓ the perturbation of the density propagates with

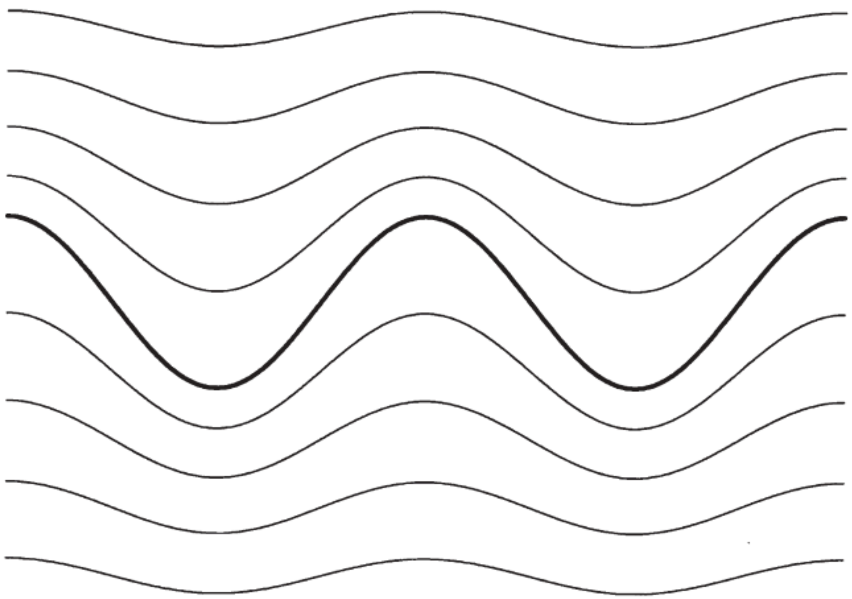
Brunt-Väisälä (buoyancy) frequency

$$\omega = \sqrt{-\frac{g}{\rho_0}\frac{\partial \rho}{\partial z}} = N$$

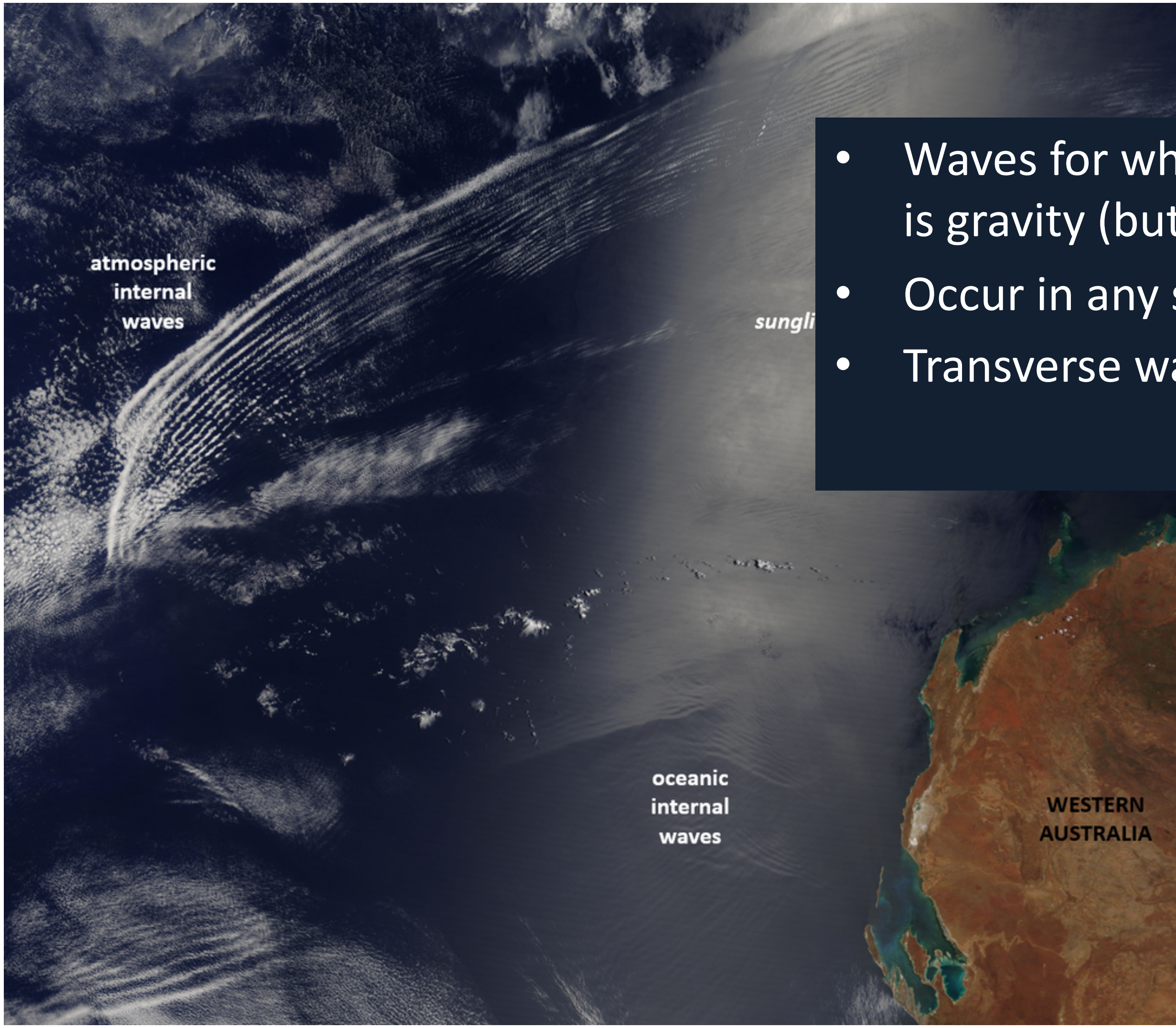
Internal gravity waves

When a body embedded into a stratified fluid oscillates, its mechanical energy is gradually transferred to the surrounding fluid:

viscous dissipation
+
internal gravity waves



Internal gravity waves

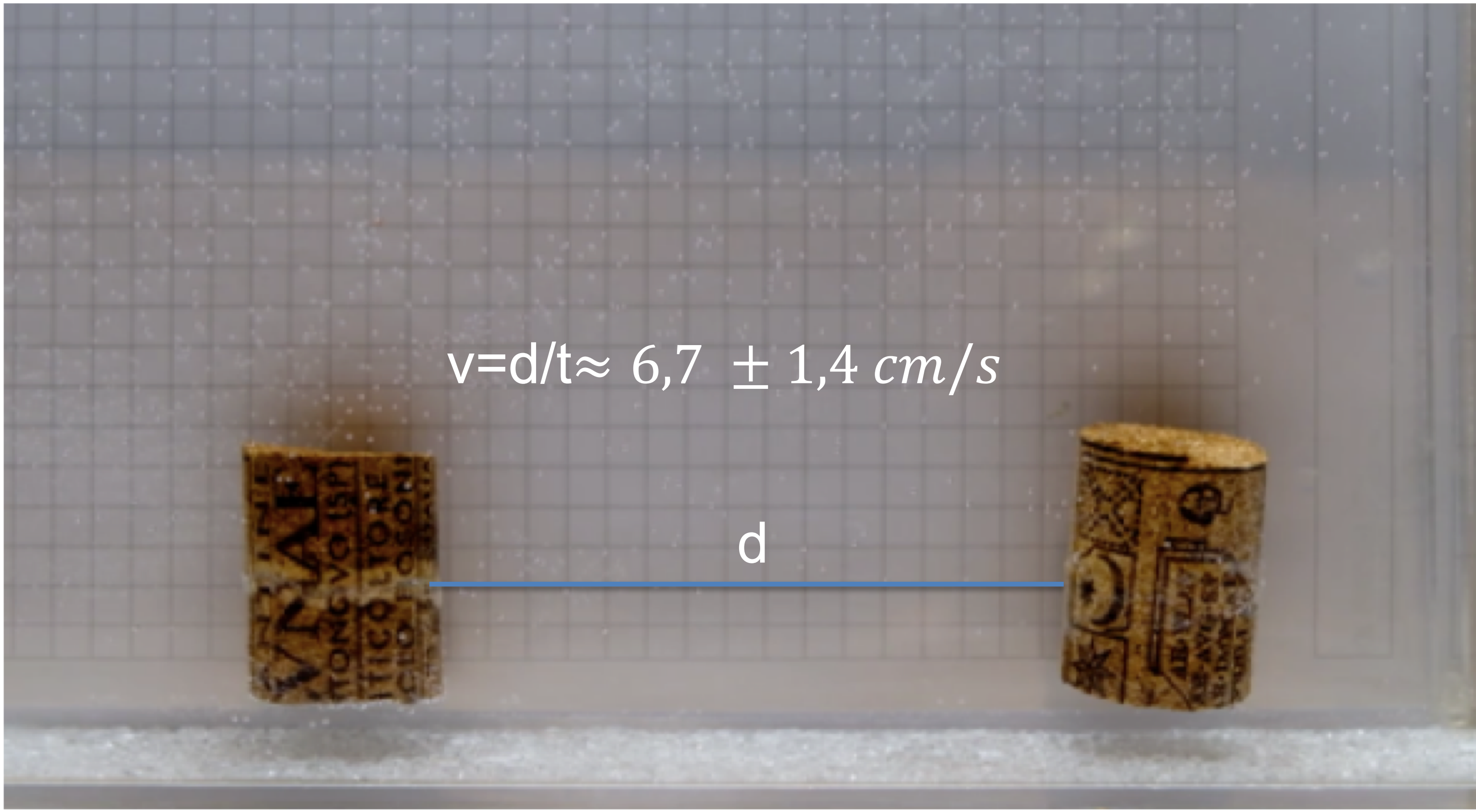


- Waves for which the restoring force is gravity (but they are not surface waves!)
- Occur in any stably-stratified region
- Transverse waves in a fluid (!)

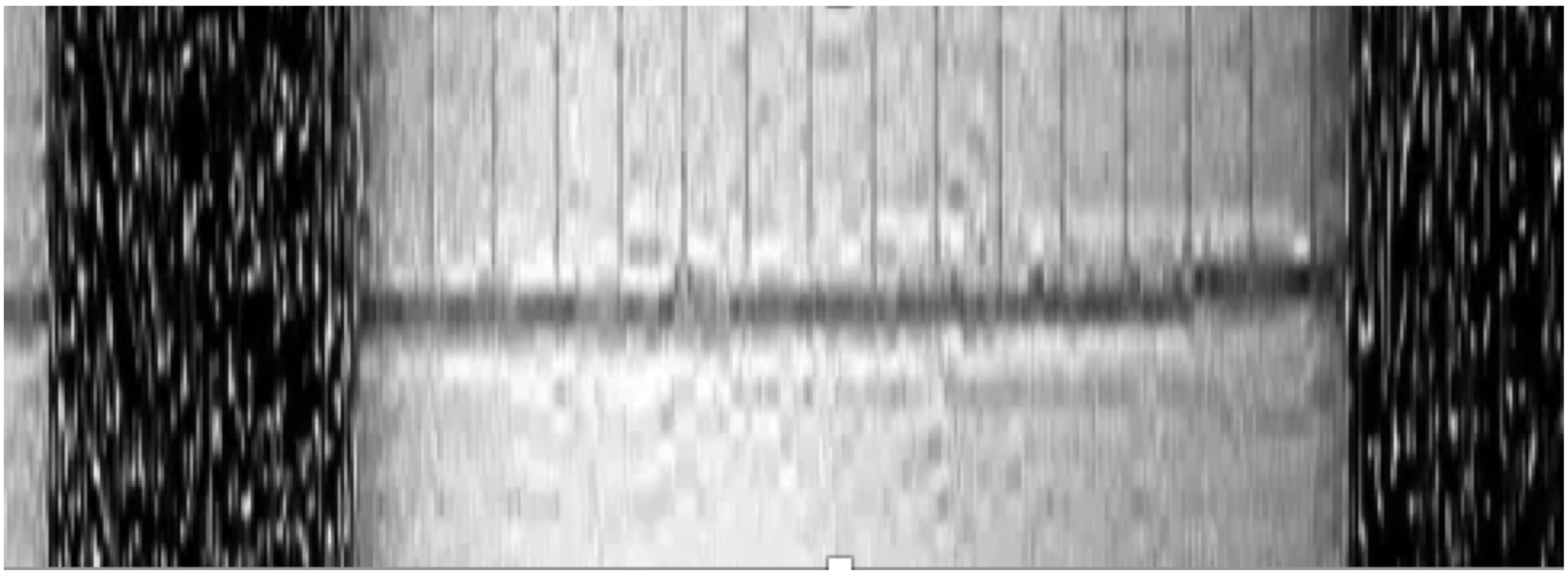
NASA credits
Satellite image Australia coast

Transfer of energy

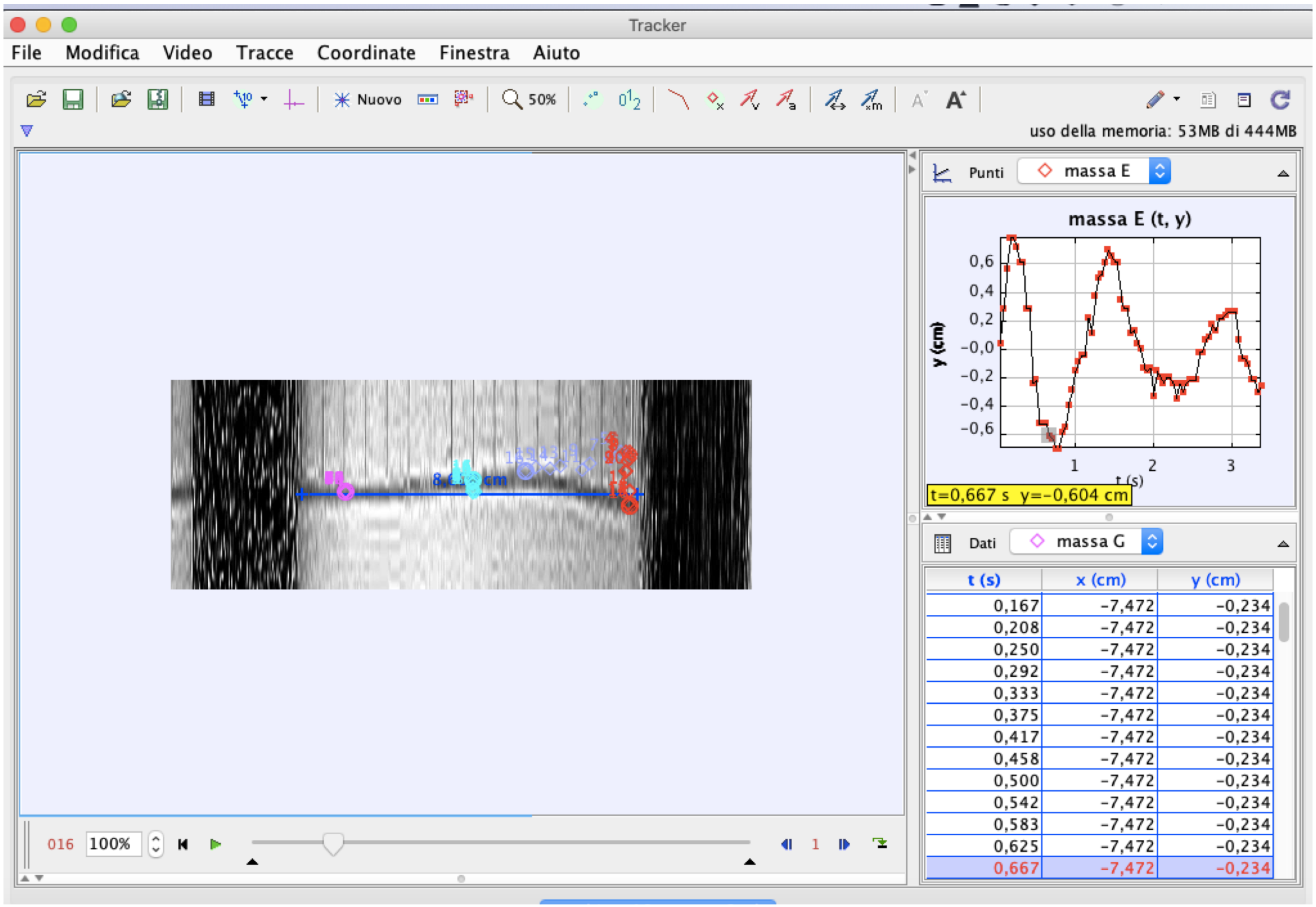
Transfer of energy



Internal gravity waves visualization



Tracking of the internal waves



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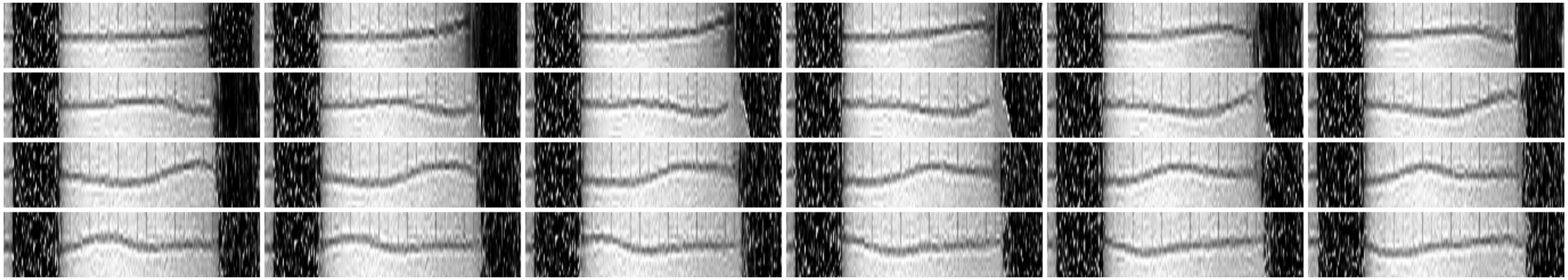
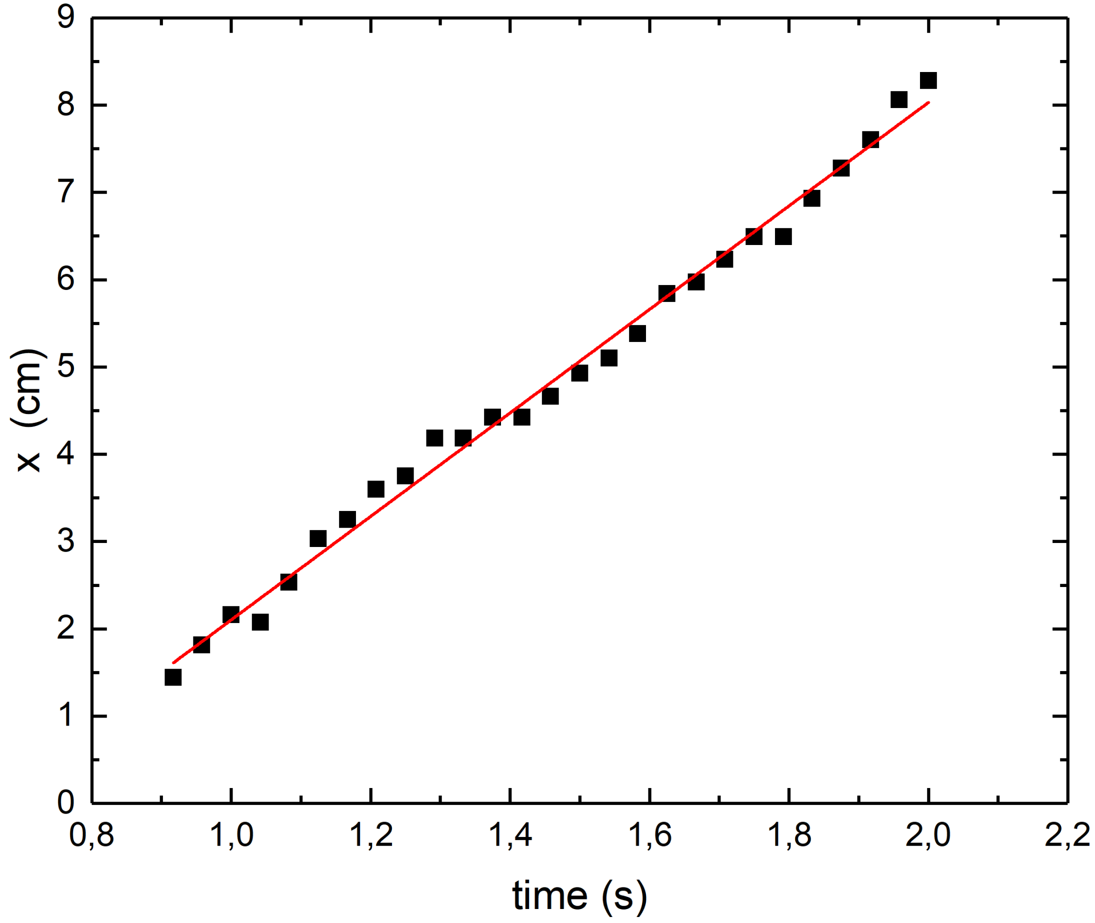


Sequence of images at frame rate 8 img s^{-1}

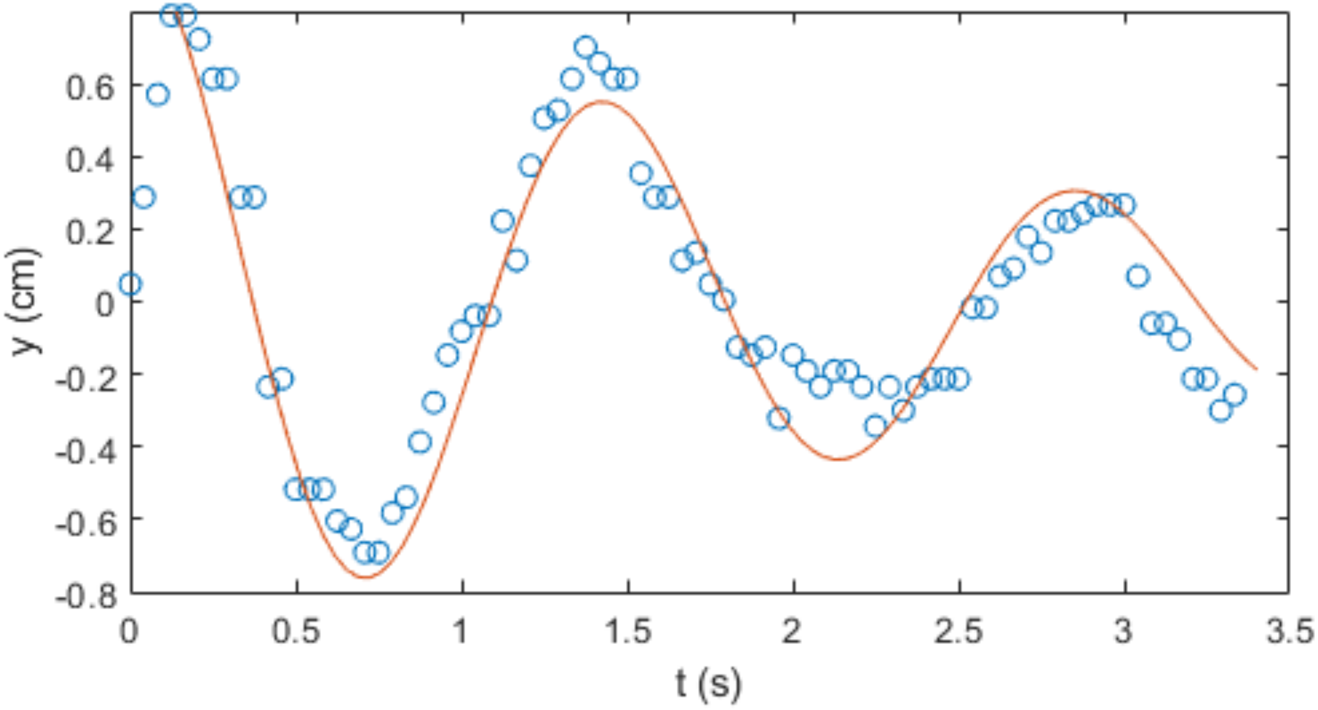
Propagation velocity



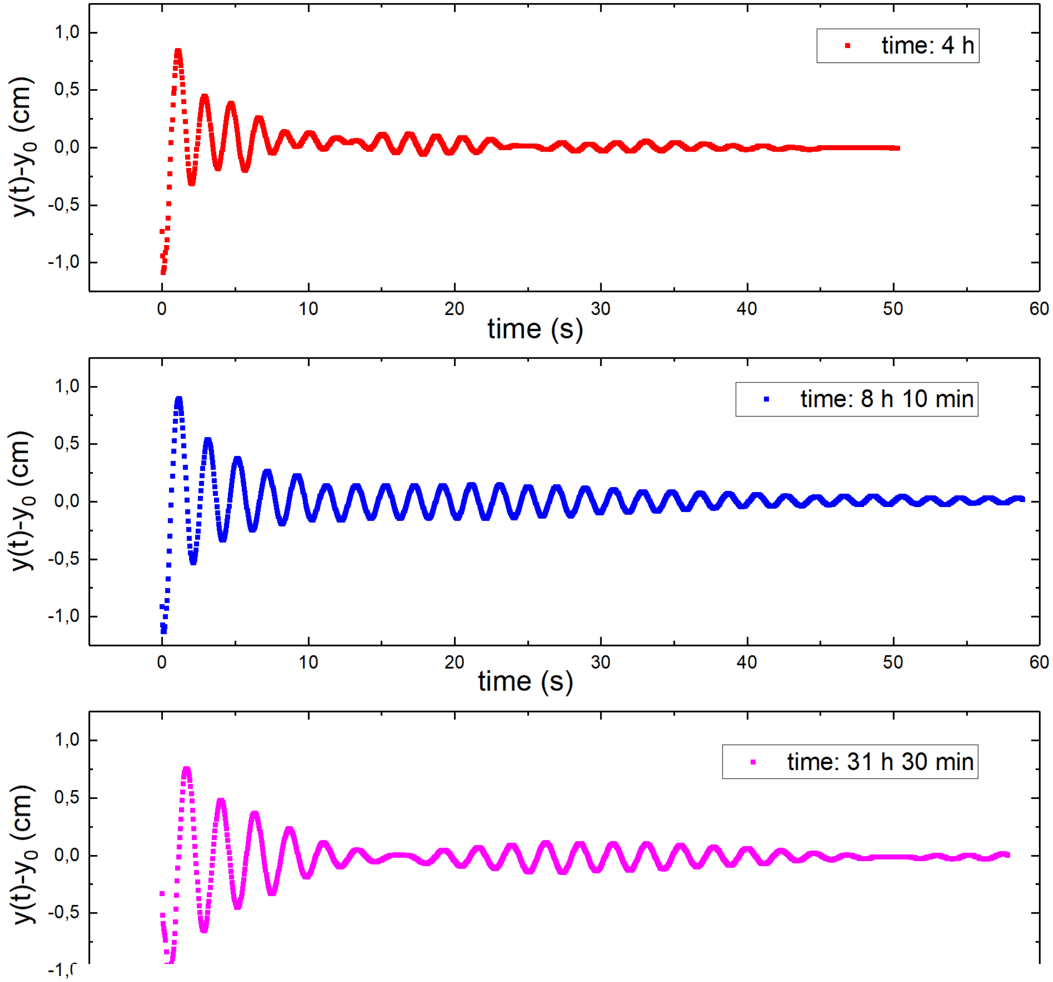
Tracking of the horizontal displacement in time of a wave peak

$$v \approx 6{,}6 \pm 0{,}6\ cm\ s^{-1}$$

Vertical displacement



Damping + beatings



Damping +

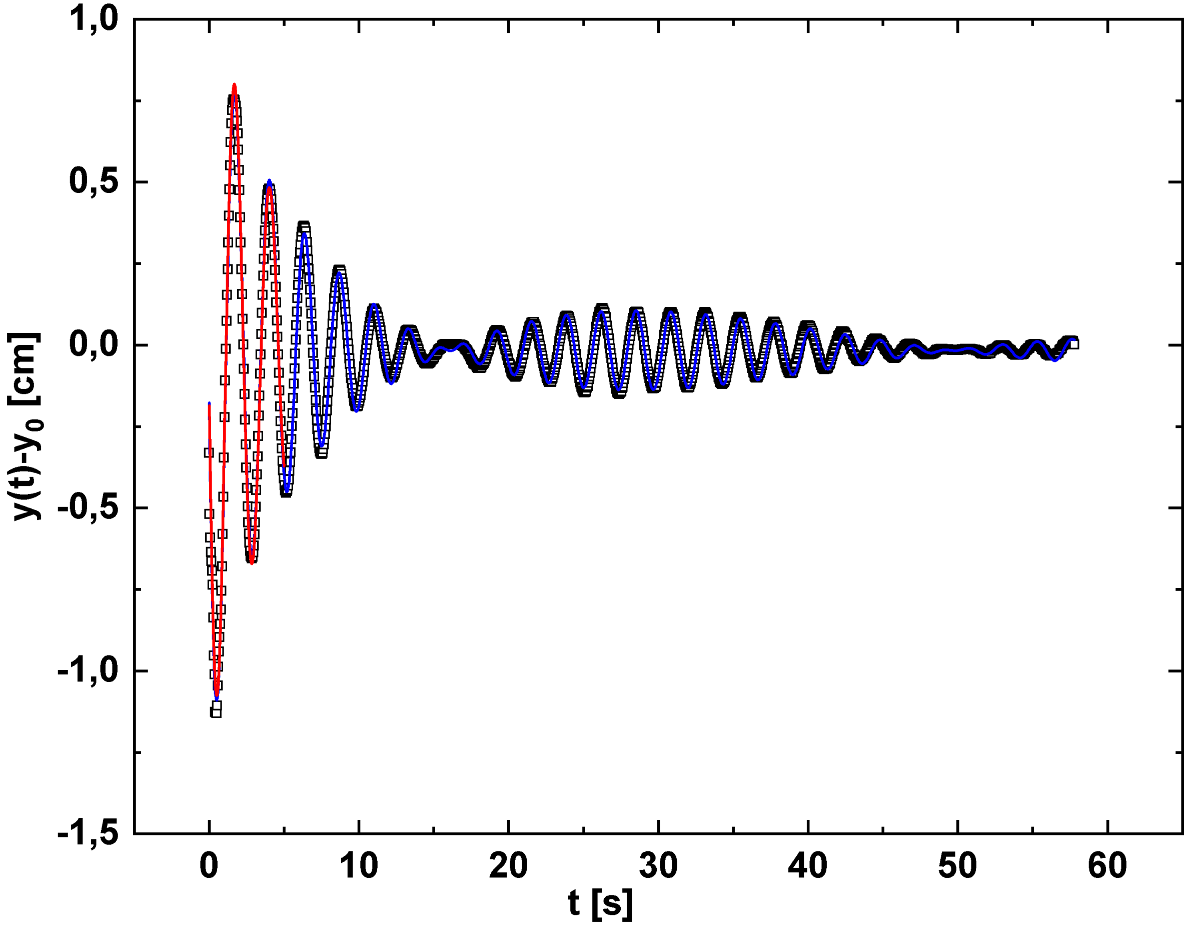
beatings between cork oscillations and back-reflected gravity wave

ω_1 Average frequency

$\Delta\omega^*$ Frequency difference

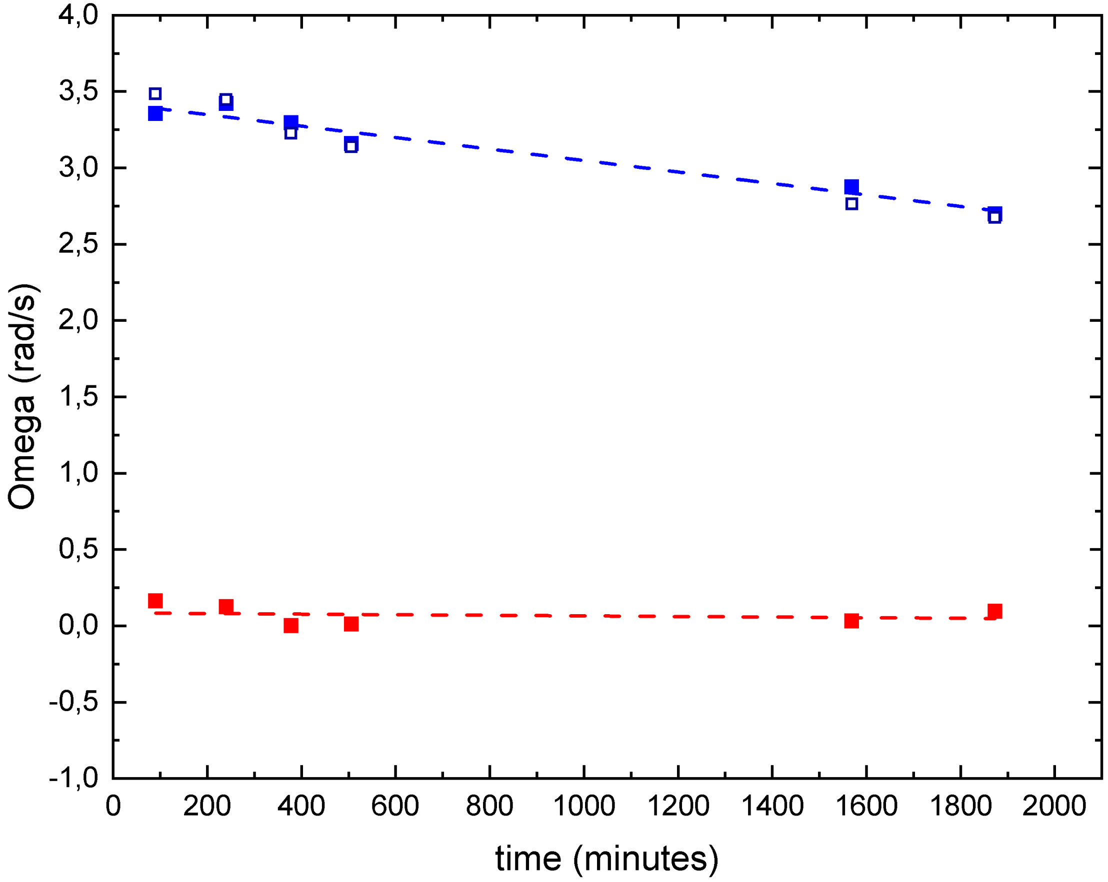
$$y(t) = A\cos(\omega_1 t + \varphi_1)\cos(\Delta\omega^* t + \varphi_2)\exp\left(-\frac{t}{\tau}\right)^{\gamma} + y_0$$

Damping + beatings



The stretched exponential term $\exp\left(-\frac{t}{\tau}\right)^{\gamma}$ with $\gamma=2$ is the best function accounting for the severe damping of the two terms of different time constant.

Frequencies in time



■ $\omega_1 \approx N = \sqrt{-\frac{g}{\rho_0}\frac{\partial \rho}{\partial z}}$

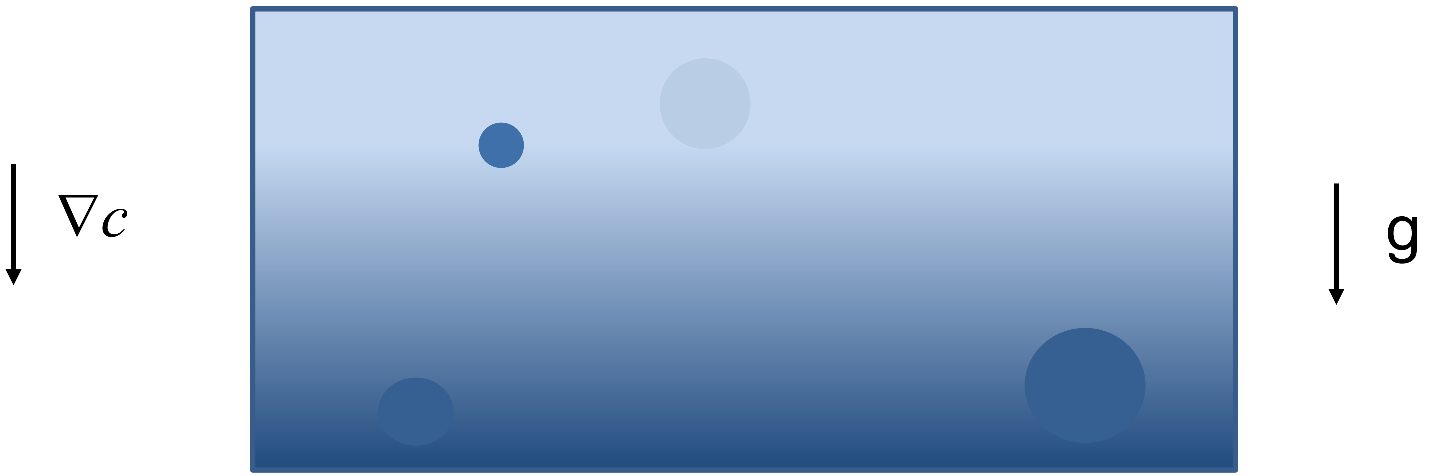
■ $\Delta\omega^*$

$$1{,}5\ \frac{rad}{s} \leq N \leq 4{,}5\ \frac{rad}{s}$$

For the salt concentration used

$$236\ \frac{kg}{m^3} \leq \frac{\partial \rho}{\partial z} \leq 2140\ \frac{kg}{m^3}$$

Non equilibrium fluctuations:
velocity fluctuations induce concentration fluctuations



Diffusion

$$\tau_{diff} = \frac{1}{Dq^2}$$

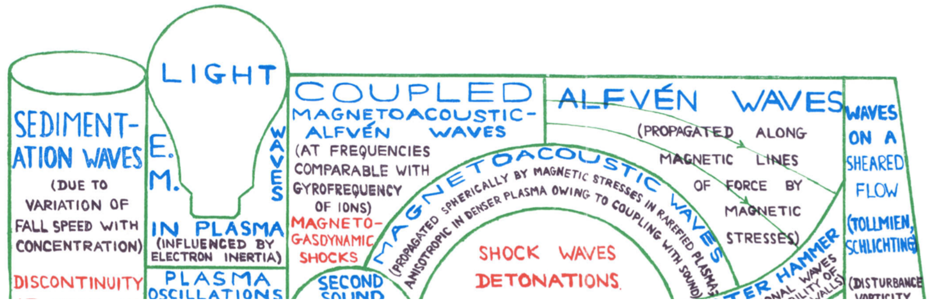
Buoyancy

$$\tau_{grav} = \frac{\upsilon q^2}{\beta g \nabla c}$$

Rolloff wave vector

$$\tau_{diff} = \tau_{grav} \quad \Rightarrow \quad q_{RO} = \sqrt[4]{\frac{\beta g \nabla c}{\nu D}}$$

Waves



Thank you for your attention

«Waves in fluids»
M.J. Lighthill Comm. On Pure and Appl, Math XX, 267 (1967)